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Generalized $^3P_0$ and $^3S_1$ Annihilation Potentials for $\bar{p}p$ Decay into Two Mesons based on a Simple Quark Model

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Abstract

Within the quark model a generalization is proposed of the commonly used annihilation potential to describe antiproton-proton annihilation into two mesons, the so-called $^3P_0$ and $^3S_1$ mechanisms. This generalized potential treats the two mechanisms in a more symmetric way, has additional angular dependence, and results in an expanded set of selection rules.

1. Introduction

As a first step on the way to find a microscopic model for the annihilation of antiproton and proton into two mesons, the process is often described in the simple quark model by diagrams representing the so-called vacuum ($^3P_0$) and one-gluon ($^3S_1$) mechanisms. A detailed description appears in a recent review of $\bar{p}p$ annihilation [1]. In particular the $^3P_0$ mechanism was inspired by earlier work on meson decay [2]. Since several years many authors have studied the $\bar{p}p$ annihilation process, prefering just one particular mechanism or some combination of $\bar{q}q$ annihilation into vacuum and one-gluon. We mention refs.[3-21], but a more complete list of references can be found in the review of ref.[1]. In general the annihilation is described by a potential

$$V_{\rm ann} \;=\; V(^3P_0) \;+\; \lambda \, V(^3S_1), \tag{1}$$

where λ is the relative strength of the two mechanisms. In the $^3P_0$ mechanism the $\bar{q}q$ pair annihilates into the effective "vacuum" state $J^\pi = 0^+$. In the $^3S_1$ mechanism the $\bar{q}q$ pair annihilates into the effective "one-gluon" state $J^\pi = 1^-$. Examples of quark diagrams for the $\bar{p}p$ annihilation into two mesons are shown in fig.1. Because quark diagrams with real gluon exchanges are infinitely more complex, the above mechanisms should be regarded as the first two leading operators in an expansion of the annihilation mechanism of the $\bar{q}q$ system into terms of increasing J^π. Since the exchanged "vacuum" and "one-gluon" are only effective this means in particular that in both cases transfer of momentum occurs , a fact that is neglected in previous descriptions of the $^3P_0$ mechanism. As a result previous treatments of $^3P_0$ and $^3S_1$ are somewhat asymmetric. Most versions of the vacuum term only contribute to $\ell_{\rm MM}$ =0 and $\ell_{\rm pp} = 1$, while the one-gluon term contributes to an entire series of angular momentum states of the $\bar{p}p$ system.

2. Generalized potential

In this Letter we propose to describe the annihilation mechanism of $\bar{p}p$ into two mesons by the potential $V_{\rm ann} = V(^3P_0) + \lambda\ V(^3S_1)$, but allowing momentum transfer in both mechanisms. The relative strength λ is treated as a parameter. Additional diagrams where two $\bar{q}q$ pairs are annihilated and another $\bar{q}q$ pair is created, should also be considered in further generalizations, in particular when annihilation into strange mesons occurs (see fig.1(c)). This can even lead to additional J^π terms in the expansion of the annihilation operator, but they are ignored at this time.

The nucleon (and antinucleon) wave function is described as a Gaussian

$$\Psi_N(\vec{r}_1,\vec{r}_2,\vec{r}_3) \ = \ N_N \ \exp\left[\ -\frac{\alpha}{2}\Sigma(\vec{r}_i-\vec{r}_N)^2\ \right] X_N \ (\text{spin, isospin, color}), \quad (2)$$

where $\vec{r}_i$ are the quark coordinates and $\vec{r}_N$ is the nucleon coordinate. An S-wave meson wavefunction is given by

$$\Phi_M(\vec{r}_1,\vec{r}_4) \ = \ N_M \ \exp\left[\ -\frac{\beta}{2}\Sigma(\vec{r}_i-\vec{r}_M)^2\right] X_M(\text{spin, isospin, color}), \quad (3)$$

where $\vec{r}_1$ and $\vec{r}_4$ are respectively the quark and antiquark coordinates, and $\vec{r}_M$ is the coordinate of the meson. Typical parameter values are $\alpha = 2.8$ fm^{-2} and $\beta = 3.23$ fm^{-2}, giving a nucleon radius of 0.60 fm and a meson radius of 0.48 fm.

The $\bar{p}p$ annihilation is then described by a non-local transition potential in terms of the relative $\bar{N}N$ coordinate $\vec{r}$, the relative two-meson coordinate $\vec{r}\,'$, and the spin-operator $\vec{\sigma}$, obtained by integrating out the unconstrained coordinates of the quarks and antiquarks.

As an example we consider from here on explicitly the reaction $\bar{p}p \rightarrow \pi^-\pi^+$, using only the diagrams 1(a) and 1(b). Here it is assumed that the

pion can be described by a quark wave function of the form of eq.(3).

3. Vacuum Mechanism

For the "vacuum" mechanism one obtains from a single diagram a potential

$$V_{\mathrm{single}}(^3P_0)(\vec{r}\,', \vec{r}\,) \sim \{A_V \; i\vec{\sigma}.\vec{r}\,' \quad + \quad B_V \; i\vec{\sigma}.\vec{r}\,\} \exp(Ar\,'^2 + Br^2 + C\vec{r}\,'.\vec{r}\,), \tag{4}$$

where A_V, B_V, A, B, and C are functions of the parameters α and β. Summing over all diagrams in case of the "vacuum" term one obtains the form

$$V_{\mathrm{total}}(^3P_0)(\vec{r}\,', \vec{r}\,) \sim \tag{5}$$
$$\{A_V \; i\vec{\sigma}.\vec{r}\,' \sinh(C\vec{r}\,'.\vec{r}\,) \quad + \quad B_V \; i\vec{\sigma}.\vec{r} \cosh(C\vec{r}\,'.\vec{r}\,)\} \exp(Ar\,'^2 \; + \; Br^2),$$

$$\text{where } A_V \; = \; \frac{\alpha(\alpha+\beta)}{2(4\alpha+3\beta)}, \tag{6}$$

$$B_V \; = \; \frac{3(5\alpha^2+8\alpha\beta+3\beta^2)}{2(4\alpha+3\beta)}, \tag{7}$$

$$A \; = \; - \; \frac{\alpha(5\alpha+4\beta)}{2(4\alpha+3\beta)}, \tag{8}$$

$$B \; = \; - \; \frac{3(7\alpha^2+18\alpha\beta+9\beta^2)}{8(4\alpha+3\beta)}, \tag{9}$$

$$C \ = \ - \ \frac{3\alpha(\alpha+\beta)}{2(4\alpha+3\beta)}. \tag{10}$$

The potential V($^3P_0$) ($\vec{r}\,'$,$\vec{r}$) is even in $\vec{r}\,'$ and odd in $\vec{r}$. It contributes to $\ell_{\pi\pi} = 0, 2, 4, \dots$, and $\ell_{pp} = 1, 3, 5, \dots$. In other words V($^3P_0$) acts in $J^\pi =$ 0^+, 2^+, 4^+, ... waves (e.g. $^3P_0$, $^3P_2$ - $^3F_2$, $^3F_4$ - $^3H_4$, ...) with isospin I = 0.

The present potential V($^3P_0$) of eq.(5), where the linear momentum of the annihilating $\bar{q}q$ pair is transfered to one of the final quarks or antiquarks, has to be compared with the standard V($^3P_0$) expression, where no linear momentum is transfered. The standard potential is

$$V' \ (^3P_0)(\vec{r}\,', \vec{r}) \ \sim \ B'_V \ i\vec{\sigma}.\vec{r} \exp(A' r'^2 \ + \ B' r^2), \tag{11}$$

where $B'_V = \alpha + \beta$, A$'$ = $-$ $\alpha/2$, and B$'$ = $-$ 3 (α + 3β)/8. The expression of eq.(5) has additional angular dependence due to the presence of the C$\vec{r}\,'.\vec{r}$ term in the exponent and the additional A_V i$\vec{\sigma}.\vec{r}\,'$ term. In previous cases (without momentum transfer) only a transition between $\ell_{\pi\pi}$=0 and ℓ_{pp}=1 was possible (e.g. $^3P_0$) , while with the present form all even J values contribute.

4. One-Gluon Mechanism

The "one-gluon" exchange term splits in a transversal part and a longitudinal part. The transversal part becomes

$$\begin{aligned} V_T(^3S_1) \ (\vec{r}\,', \vec{r}) \quad &\sim \\ \{A_T \ i\vec{\sigma}.\vec{r}\,' \cosh(C\vec{r}\,'.\vec{r}) \quad &+ \quad B_T \ i\vec{\sigma}.\vec{r} \sinh(C\vec{r}\,'.\vec{r})\} \exp(A r'^2 + B r^2) \end{aligned} \tag{12}$$

where A, B, and C are the same as in eqs.(8-10) , while

$$A_T \;=\; -\;\frac{2\alpha(\alpha+\beta)}{4\alpha+3\beta}, \tag{13}$$

$$B_T \;=\; -\;\frac{3\alpha(\alpha+\beta)}{4\alpha+3\beta}. \tag{14}$$

The potential $V_T(^3S_1)(\vec{r}\,',\vec{r})$ is odd in $\vec{r}\,'$ and even in $\vec{r}$, and contributes to $\ell_{\pi\pi} = 1, 3, 5, \ldots$, and $\ell_{pp} = 0, 2, 4, \ldots$. Therefore $V_T(^3S_1)$ acts in $\bar{p}p$ states with $J^\pi = 1^-, 3^-, 5^-, \ldots$ waves (e.g. $^3S_1 -^3 D_1$, $^3D_3 -^3 G_3$, $^3G_5 -^3 I_5$, $\ldots$) with isospin I = 1. The basic form of $V_T(^3S_1)$ is the same as used before in the literature and is described for example in ref.[5].

The second "one-gluon" term is longitudinal and can be written as

$$\begin{aligned} V_L(^3S_1)\ \ (\vec{r}\,',\vec{r}) \quad &\sim \\ \{A_L\ i\vec{\sigma}.\vec{r}\,'\sinh(C\vec{r}\,'.\vec{r}) \ \ &+\ \ B_L\ i\vec{\sigma}.\vec{r}\cosh(C\vec{r}\,'.\vec{r})\}\exp(Ar'^2+Br^2), \end{aligned} \tag{15}$$

$$\text{where } A_L \;=\; -\;\frac{\alpha(\alpha+\beta)}{4\alpha+3\beta}, \tag{16}$$

$$B_L \;=\; \frac{9(\alpha+\beta)^2}{2(4\alpha+3\beta)} \tag{17}$$

$V_L(^3S_1)$ has the same symmetry as V$(^3P_0)$ of eq.(5) and therefore acts in the same J^π waves. The parameters A, B, and C in eq.(15) are again the same as eqs.(8-10).

5. Concluding Remarks

The generalized potential $V_{\rm ann} = V(^3P_0) + \lambda V(^3S_1)$ is a function of the relative two-meson coordinate $\vec{r}\,'$, the relative $\bar{p}p$ coordinate $\vec{r}$, and the spin-operator $\vec{\sigma}$. All coefficients are functions of the nucleon and meson size-parameters α and β. The various parts of $V_{\rm ann}$ satisfy systematic selection rules. Both vacuum and one-gluon exchange mechanisms are treated on equal footing in a symmetric way. Both can contribute in a series of J^π channels.

From the point of view of the range of this potential in either variable $\vec{r}$ or $\vec{r}\,'$, it is interesting to mention typical values of A, B, and C. With the values adopted above for α and β one finds A = -1.80 fm^{-2}, B = -5.59 fm^{-2}, and C = - 1.21 fm^{-2}, while for the potential V$'$ $(^3P_0)$ of eq.(11) (where there is no momentum transfered), the corresponding values are A$'$ = -1.40 fm^{-2}, B$'$ = - 4.68 fm^{-2}, and C$'$ = 0.

The above approach can be extended to include further diagrams (e.g. fig.1(c)). It can be applied to $\bar{p}p$ annihilation into other mesons, including strange mesons or into strange baryons such as $\bar{\Lambda}\Lambda$. It may also be interesting to see whether these generalized $^3P_0$ and $^3S_1$ potentials alter predictions of the branching ratios for the various $\bar{p}p$ annihilations.

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FIGURE CAPTIONS

FIG. 1. Quark diagrams corresponding to $\bar{p}p$ annihilation into two mesons.